

MATH3290 Mathematical Modeling 2018/2019
Assignment 1
Due Date: 5pm, Oct 2

Note: Answer all three problems. Turn in the assignment in the assignment box located in LSB.

1. Founded in 1998, Google is a multinational internet service corporation headquartered in California, United States. Some investors are interested in the future value of Google. The data in Table 1 displays Google's revenue worldwide in the past ten years. Find a dynamical system model that fits the data fairly well. Use your model to predict Google's revenue worldwide in 2018 and 2019 for the investors. In the process of model development, state your arguments clearly.

Year	Revenue in billion U.S. dollars
2008	21.80
2009	23.70
2010	29.30
2011	37.90
2012	50.18
2013	55.51
2014	65.67
2015	74.54
2016	89.46
2017	109.65

Table 1: Google's revenue worldwide from 2008 to 2017 (Source: Google).

2. Suppose you have just been hired as a brand manager of toothpaste for a large company in Hong Kong. Your job mainly involves maximizing the sales of the products. Table 2 records the figure of your company in the past 30 months. You are asked to select appropriate factors that may contribute to your sales and formulate a model that simulate your sales. In the process of model development, state your arguments clearly.

Price (\$)	Competitors' Price (\$)	Advertising Fee (<i>million</i> \$)	Price Difference (\$)	Sales (<i>million</i> \$)
3.85	3.80	5.50	-0.05	7.38
3.75	4.00	6.75	0.25	8.51
3.70	4.30	7.25	0.60	9.52
3.70	3.70	5.50	0.00	7.50
3.60	3.85	7.00	0.25	9.33
3.60	3.80	6.50	0.20	8.28
3.60	3.75	6.75	0.15	8.75
3.80	3.85	5.25	0.05	7.87
3.80	3.65	5.25	-0.15	7.10
3.85	4.00	6.00	0.15	8.00
3.90	4.10	6.50	0.20	7.89
3.90	4.00	6.25	0.10	8.15
3.70	4.10	7.00	0.40	9.10
3.75	4.20	6.90	0.45	8.86
3.75	4.10	6.80	0.35	8.90
3.80	4.10	6.80	0.30	8.87
3.70	4.20	7.10	0.50	9.26
3.80	4.30	7.00	0.50	9.00
3.70	4.10	6.80	0.40	8.75
3.80	3.75	6.50	-0.05	7.95
3.80	3.75	6.25	-0.05	7.65
3.75	3.65	6.00	-0.10	7.27
3.70	3.90	6.50	0.20	8.00
3.55	3.65	7.00	0.10	8.50
3.60	4.10	6.80	0.50	8.75
3.65	4.25	6.80	0.60	9.21
3.70	3.65	6.50	-0.05	8.27
3.75	3.75	5.75	0.00	7.67
3.80	3.85	5.80	0.05	7.93
3.70	4.25	6.80	0.55	9.26

Table 2: Toothpaste sales in 30 months.

3. In stock market, there are generally three trends:

Bull markets: periods of time where prices generally are rising, due to the actors having optimistic hopes of the future.

Bear markets: periods of time where prices generally are declining, due to the actors having a pessimistic view of the future.

Stagnant markets: periods of time where the market is characterized by neither a decline nor rise in general prices.

In fair markets, it is assumed that the market information is distributed equally among its actors and that prices fluctuate randomly. This means that every actor has equal access to information such that no actor has an upper hand due to inside-information. Through technical analysis of historical data, certain patterns can be found as well as their estimated probabilities. For example, consider a hypothetical market where historical data has given us the following patterns. After a week characterized of a bull market trend, there is a 90% chance that another bullish week will follow. Additionally, there is a 7.5% chance that the bull week instead will be followed by a bearish one, or a 2.5% chance that it will be a stagnant one. After a bearish week theres an 80% chance that the upcoming week also will be bearish, and so on. Table 3 states the probabilities of transition from one market to another market.

	Bull	Bear	Stagnant
Bull	0.900	0.075	0.025
Bear	0.150	0.800	0.050
Stagnant	0.250	0.250	0.500

Table 3: Transition probability.

- (a) Formulate a model for this problem using a system of difference equations. State clearly the meanings of the notations in the model.
- (b) Suppose we are now in a bearish week. Using the model in (a), calculate the possibilities of a bull, bear or stagnant week for the following 10 weeks.
- (c) Find the equilibrium point, and determine its stability.